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Standard Model Predictions for Weak Boson Pair Production in e^-e^- Scattering

F. Cuypers[a], K. Kołodziej[b,1,2] and R. Rückl[a,b,1]

[a]*Max-Planck-Institut für Physik, Werner-Heisenberg-Institut,
Föhringer Ring 6, D–80805 München, FRG*

[b]*Sektion Physik der Universität München,
Theresienstr. 37, D–80333 München, FRG*

Abstract

We study polarized e^-e^- scattering into $W^-W^-\nu_e\nu_e$, $W^-Z^0e^-\nu_e$ and $Z^0Z^0e^-e^-$ final states within the framework of the standard model of electroweak interactions. These mechanisms for producing pairs of weak gauge bosons are potential backgrounds to new reactions beyond the realm of the standard model. At a centre-of-mass energy of 500 GeV the total cross sections are calculated to be 2.5, 9.4 and 1.0 fb, respectively, for unpolarized beams. The energy behaviour of the cross sections as well as particularly interesting differential distributions are presented and the topology of the final states is discussed.

1 Introduction

It is becoming clear that a linear collider will be one of the most promising tools for high energy experimentation in the begin of the next century. A number of world-wide collaborations is already developing different designs aiming at a 0.5 – 2 TeV machine capable of delivering 10 fb^{-1} of luminosity per year. The possibility of operating a linear collider in e^+e^-, e^-e^-, $e\gamma$ or $\gamma\gamma$ modes [1], with high degrees of polarization [2], is one of the big advantages of this new facility.

An interesting feature special to e^-e^- reactions is their particular sensitivity to lepton-number violating processes. For example, a resonance in the channel $e^-e^- \rightarrow \mu^-\mu^-$ would reveal the presence of dileptons [3], *i.e.* doubly charged gauge bosons which appear naturally in a wide class of gauge extensions of the standard model . Another very important example is the observation of W-pair production in $e^-e^- \rightarrow W^-W^-$ which would provide strong evidence for the existence of Majorana neutrinos [4]. Furthermore, the production of chargino or selectron pairs via sneutrino or neutralino exchanges in $e^-e^- \rightarrow \tilde{\chi}^-\tilde{\chi}^-$ or $e^-e^- \rightarrow \tilde{e}^-\tilde{e}^-$ may serve as one of the most powerful probes for supersymmetry [5, 6].

But also processes which do not violate lepton number conservation can be of interest in e^-e^- collisions. Indeed, the reaction $e^-e^- \rightarrow e^-W^-\nu_e$ provides informations on possible anomalous boson couplings which are complementary to those obtainable in e^+e^-, $e\gamma$ and $\gamma\gamma$ scattering [7]. Furthermore, the existence of an extra Z' gauge boson can be better probed in Møller scattering than with e^+e^- collisions [8].

In order to evaluate this new physics potential it is necessary to understand the standard model processes very accurately. We have begun to study this problem in a recent publication [9], where we have reported results on the total cross section and important differential distributions for the reaction $e^-e^- \rightarrow W^-W^-\nu_e\nu_e$. Here we extend this study to $W^-Z^0e^-\nu_e$ and $Z^0Z^0e^-e^-$ final states and provide a comprehensive account of our results on pair production of weak gauge bosons in e^-e^- scattering. Obviously these reactions are potential sources of background to the exotic reactions mentioned above, if the Z^0 bosons decay invisibly or electrons are lost along the beam-pipe. In addition, gauge boson pair production may also become interesting in its own right in the TeV energy regime, because of subtle $SU(2)_L \times U(1)_Y$ gauge cancellations which depend on the quartic gauge coupling of W bosons. Preliminary results of our study have been reported at a recent workshop [10].

The paper is organized as follows. In Section 2 we describe some technicalities of the calculation of the helicity matrix elements and the phase space integration. The total cross sections and events rates in the energy range from 500 GeV to 2 TeV are given in Section 3. In Section 4 we present some important differential distributions and discuss the general topology of the final states. We also indicate kinematical cuts which would eliminate these standard model processes as backgrounds to 'new' physics. A brief summary and conclusions are given in Section 5.

2 Helicity Amplitudes and Phase Space Integration

In Figs **??** to **??** we display the different topologies of the lowest order Feynman diagrams for the processes

$$e^-(p_1) + e^-(p_2) \rightarrow \begin{cases} W^-(k_1) + W^-(k_2) + \nu_e(q_1) + \nu_e(q_2) \; , & (1) \\[2mm] W^-(k_1) + Z^0(k_2) + e^-(q_1) + \nu_e(q_2) \; , & (2) \\[2mm] Z^0(k_1) + Z^0(k_2) + e^-(q_1) + e^-(q_2) \; , & (3) \end{cases}$$

where the particle momenta are indicated in parenthesis. The actual diagrams can be easily obtained from the generic ones given in the figures by an appropriate particle assignment and by permutations of the momenta of identical particles. In the limit of zero electron mass (when the electron does not couple to the Higgs field), there are 66 diagrams contributing to the $W^-W^-\nu_e\nu_e$ final state. Since in this case each of the two fermion lines is coupled to at least one W^- boson, as can be seen in Fig. **??**, only the left-left (LL) combination of initial polarizations has a non-zero cross section. In contrast, the $W^-Z^0e^-\nu_e$ final states can be produced with LL and left-right (LR) beams. Here one has contributions from 88 and 37 Feynman diagrams, respectively. Finally, in the $Z^0Z^0e^-e^-$ channel all combinations of beam polarizations have finite cross sections. The number of Feynman diagrams for LL, LR and right-right (RR) scattering is 86, 43 and 86, respectively.

The large number of diagrams makes the use of conventional trace techniques for calculating the matrix element squared rather uneconomical. Moreover, in the high energy regime large gauge cancellations can cause numerical instabilities when squares of diagrams and interference terms are added. It is therefore advisable to compute directly the helicity amplitudes associated with definite polarizations of the initial and final states. In order to warrant to a high degree the correctness of our results, we have performed two independent calculations, making use of two different four-body phase space integration routines and helicity amplitude formalisms. The first helicity method is the conventional one, based on the Weyl–van der Waerden formalism. Although originally developed for massless particles, it was extended to massive bosons in Ref. [11]. The second method, described in detail in Ref. [12], uses the Weyl representation of the Dirac matrices and spinors, which is particularly suitable for handling massless fermions. In this representation the spinors have only two non-zero entries and the Dirac matrices have an off-diagonal form, when represented by blocks of 2×2 matrices. This allows to reduce the 4×4 Dirac algebra to the 2×2 Pauli algebra. Given the initial and finite state polarization, one has to calculate the relevant elements of a 2×2 matrix for each Feynman diagram. This matrix is composed of an odd number of Pauli matrices contracted with some combinations of the particle momenta or the gauge boson polarization vectors. For better numerical performance, the latter are taken to be real. The results of the two calculations are in perfect numerical agreement.

In addition, for each of the processes (1) to (3) we have checked internal or external gauge invariance. All calculations have been performed in a general covariant (R_ξ) gauge. For

$e^-e^- \to W^-W^-\nu_e\nu_e$ we verified gauge invariance with respect to a change of the gauge parameter ξ in the gauge boson propagator. Numerically we have observed invariance of the matrix element squared averaged over polarizations up to 12 digits, when the gauge parameter ξ is varied over the range $-10^5 \le \xi \le 10^5$. For $e^-e^- \to W^-Z^0e^-\nu_e$ and $e^-e^- \to Z^0Z^0e^-e^-$ with the Z^0 boson replaced by a photon, we have checked external gauge invariance. This time we have observed numerically a drop of the real and imaginary part of the matrix elements by several orders of magnitude when the photon polarization vector is replaced by its momentum.

The electron mass is neglected everywhere, except in the denominator of the photon propagator whenever it is needed to regulate collinear singularities. For instance, in reaction (2), when a photon is coupled to an on-shell electron line, the momentum transfer flowing through the photon propagator can be written as

$$t = (p_1 - q_1)^2 = -\sqrt{s}E'\beta\beta'(1 - \cos\theta) - t^{\min} \, , \tag{4}$$

where the particle momenta are defined in Eq. (2). E' is the energy of the final state electron, $\beta = (1 - \frac{4m_e^2}{s})^{\frac{1}{2}}$ and $\beta' = (1 - \frac{m_e^2}{E'^2})^{\frac{1}{2}}$ are the velocities of the initial and final state electrons, and θ is the angle of the final state electron with respect to the beam axis in the centre-of-mass system. To order m_e^2, the minimal absolute value of the momentum transfer is given by

$$t^{\min} = m_e^2 \frac{(\sqrt{s} - 2E')^2}{2\sqrt{s}E'} \, . \tag{5}$$

To avoid the collinear singularity in the phase space integral of terms proportional to $1/t$ in the squared matrix element, one has to keep term (5) in the denominator of the photon propagator given by Eq. (4). This approximate procedure takes properly into account the leading collinear logarithms, but neglects constant terms. The point is that in addition to the terms proportional to $1/t$ there appear m_e^2/t^2-terms in the squared matrix elements which give rise to constant contributions after integration over the collinear phase space regions. These contributions are neglected by the above procedure. However, since these constant terms are not enhanced in any way with respect to the leading logarithmic terms, we estimate the accuracy of the approximation to be at the per cent level, which seems to be good enough for most purposes. Similarly, terms proportional to m_e^2 in the amplitude squared of reaction (2) also yield small yet finite cross sections for the RR polarization. Indeed, collinear electrons have some probability to undergo a helicity flip [13]. For unpolarized beams, this effect is expected to be of the order of the neglected terms proportional to m_e^2/t^2, discussed above. As the photon in these collinear configurations approaches its mass-shell, the corresponding contributions can readily be estimated within the Weizsäcker-Williams approximation [14]. Note that there are no singularities in the $W^-W^-\nu_e\nu_e$ channel, since the internal photon never couples to an on-shell fermion line. In this case, the predictions made neglecting the electron mass should be perfectly accurate. Also, the potential collinear divergences of W^- or Z^0 radiation are automatically regulated by the W^- or Z^0 boson mass.

3

In order to obtain better numerical convergence of the Monte Carlo integration we introduce the variable

$$y = \frac{1}{2} \ln \frac{1 + 2\Delta + \cos\theta}{1 + 2\Delta - \cos\theta} \ , \tag{6}$$

where Δ is an appropriate regulator and θ is the angle between the final state particle in question and the beam axis [13]. For the final electrons in reaction (2) and (3) we find

$$\Delta = \frac{m_e^2}{s} \tag{7}$$

to be a good choice, while for the Z^0 bosons of reaction (3) we take

$$\Delta = \frac{m_Z^2}{s} \ . \tag{8}$$

Using Eq.(6), the collinear factor $(1 - \cos\theta)$ on the right-hand-side of Eq. (4) is replaced by

$$1 - \cos\theta = 2\frac{1 + 2\Delta}{1 + \exp(-2y)} - 2\Delta \ . \tag{9}$$

The new variables y and the remaining variables which are not discussed here are generated according to flat distributions. In this way we obtain stable results.

3   Production Rates

In this section, we present our results on polarized total cross sections for the reactions (1) to (3). The relevant input parameters used in our calculation are given below. For the Z^0 and W^- boson masses we take $m_Z = 91.19$ GeV and $m_W = 80.3$ GeV. The corresponding value of the weak mixing angle is given by $\sin^2\theta_W = 0.225$. Furthermore, for the fine structure constant we take its value at the Z^0 pole, that is $\alpha(m_Z^2) = 1/128.87$ [15]. This choice of scale is rather arbitrary, as usual in t-channel processes. Since one is dealing with $O(\alpha^4)$ processes the resulting uncertainty is of the order of several per cent. It is expected to exceed the errors due to the neglect of the electron mass, pointed out in the previous section. The value adopted for the Higgs boson mass is $m_H = 100$ GeV. Its exact value turns out to be irrelevant for reactions (1) and (2). The same is true for reaction (3), provided $m_H < 2m_Z$. If $m_H > 2m_Z$ diagram 4 of Fig. **??** develops a resonance which is damped by the appropriate Higgs width. We have not considered this possibility here.

The numerical predictions at a centre-of-mass energy of 500 GeV for polarized and unpolarized beams are collected in Table 1. For a realistic integrated luminosity of 10 fb^{-1} and unpolarized beams, one can expect about 25 $W^-W^-\nu_e\nu_e$, 100 $W^-Z^0e^-\nu_e$ and 10 $Z^0Z^0e^-e^-$ events. From the leptonic and hadronic branching ratios of the W^-, Z^0 and τ, one can easily

estimate the total number of events with a given number of neutrinos, charged leptons and hadronic jets. In this context, though, it is important to note (*cf.* next section) that most of the final state leptons not coming from W^- or Z^0 decays disappear along the beam-pipe, whereas most of the decay products of the (slow and rather isotropic) gauge bosons should be observable. For some important final states, the number of events from reactions (1-3) and some possible sources beyond the standard model are listed in Table 2, assuming 10 fb^{-1} of accumulated luminosity and unpolarized beams. It is of course straightforward to perform the same exercise with polarized beams. As we shall show in the next section, one can reduce the standard model background dramatically by some simple kinematical cuts.

The energy dependence of the polarized cross sections is illustrated in Fig. **??**. Beyond $\sqrt{s} = 1$ TeV the cross sections rise almost linearly. Whereas for centre-of-mass energies above 1 TeV the cross sections for reactions (1) and (2) become sizeable, the cross sections of reaction (3) remain much smaller over the whole energy range considered. The striking differences in the relative magnitudes of the cross sections apparent from Fig. **??** result from the differencies in boson-lepton couplings, the absence of triple and quartic gauge boson couplings in (3) and the presence of photon singularities in reactions (2) and (3). For example, we have checked that imposing an angular cut, which eliminates configurations in which final state leptons are collinear with the beam axis, diminishes the cross section of reaction (2) to much larger extent than the cross section of reaction (1).

To conclude this section, we stress that important cancellations take place between different gauge invariant subsets of Feynman diagrams. At $\sqrt{s} = 2$ TeV the total cross section can be three orders of magnitude smaller than the individual contributions of some sets of graphs. At higher energies, the eventually ensuing numerical instabilities might require the use of new calculational methods or approximations.

4   Differential Distributions

With the help of the relevant differential distributions, we now characterize the typical topology of the final states of reactions (1) to (3), and examine which kinematical cuts can suppress them most efficiently. For definitness we concentrate on the process $e_L^- e_L^- \to W^- Z^0 e^- \nu_e$ in the low ($\sqrt{s} = 500$ GeV) and the high ($\sqrt{s} = 2$ TeV) energy regimes. The total cross sections are respectively 23 fb and 660 fb. When a comparison is applicable, the leptons and gauge bosons emerging from the other processes have very similar behaviour.

An important feature of all reactions is the strong tendency of the final leptons to emerge back-to-back and close to the beam-pipe. This is particularly true for the electrons in the processes (2) and (3), because of the coupling to low-virtuality photons. Moreover, with increasing collision energy this collinearity becomes more and more extreme.

The angular distributions of the gauge bosons, shown in Fig. **??**, for the W^-, are also peaked along the beam axis but by far not as strongly. At high energies, the shape of the distribution remains essentially unchanged, except for an enhancement in the small angle region. The Z^0 distribution is only slightly more isotropic. Obviously, for the asymmetric combination of beam polarization LR, the angular distributions are also asymmetric. To be specific, the final state electron and W^- distributions peak in the direction of the right-handed beam, while the neutrino and Z^0 distributions peak in the opposite directions. However, at high energies the W^- and Z^0 distributions become more symmetric.

In Fig. **??**, we plot the distributions of the angles spanning the W^- and the Z^0, and the W^- and the final electron. Because the Z^0 couples stronger to the neutrino than to the electrons, it is mainly bremsstrahled off the outgoing neutrino line. Therefore, the Z^0 tends to be emitted parallel to the neutrino, *i.e.* anti-parallel to the electron. To conserve momentum, the W^- must then be emitted in the hemisphere of the electron. While at high energies this peaking becomes more pronounced, one starts to observe at the same time configurations with a small angle W^-Z^0-pair. The e^-W^- correlation is very similar to the one predicted for the reaction $e^-e^- \rightarrow e^-W^-\nu_e$ [7].

The energy distributions of the electron and the W^- are displayed in Fig. **??**. Again, as in $e^-e^- \rightarrow e^-W^-\nu_e$, irrespective of the centre-of-mass energy the W^- has a strong tendency to be emitted with moderate kinetic energy. In contrast, the electron, and even more so the neutrino, have high kinetic energies. In fact, at large centre-of-mass energies the final leptons carry away most of the beam energy. At these large energies, when the boson masses become negligible, the energy distributions follow roughly the familiar Bremsstrahlung spectra, *i.e.* $1/x$ for the gauge bosons and $1/(1-x)$ for the leptons.

Since hadronic decays of the gauge bosons make up the bulk of the events (*c.f.* Table 2), it may be interesting to describe some typical hadronic observables. This is done in Fig. **??**, where the total hadronic energy and transverse momentum distributions are displayed side by side. Independently of the centre-of-mass energy, the transverse momentum remains peaked close to the value $p_\perp = m_{W,Z}$. As expected from the shapes of the angular distributions, this confirms that the two gauge bosons are preferentially emitted in opposite hemispheres. The energy fraction carried by the massive gauge bosons is of course substantial at low collision energy, but shrinks noticeably as the centre-of-mass energy increases. Nevertheless, even at $\sqrt{s} = 500$ GeV the tail of the energy distribution of the gauge boson pair dies out rapidly in the upper energy range. Consequently, imposing a lower limit on the total hadronic energy, say $E_{\text{hadrons}} > 0.8\sqrt{s}$, would be very effective in reducing the event rate to an unobservable level. On the other hand, such a generous cut should not influence the number of hadronic events expected from direct W^-W^- production via the t-channel exchange of a Majorana neutrino [4], even if one takes into account initial Bremsstrahlung. Similarly, requiring the transverse momentum not to exceed the experimental resolution would fullfil the same purpose.

These observations about the angular and energy distributions of the leptons and gauge bosons, allow us to characterize roughly the event topologies of processes (1) to (3) as follows: the leptons carry away a major portion of the energy along the beam directions, while the gauge bosons are emitted with small velocity and more isotropically. In the case of reaction (2), the W^- is emitted preferentially in the hemisphere of the electron, while the Z^0 is to be found more often on the neutrino side.

5 Conclusions

We have performed a detailed analysis of three important mechanisms for weak gauge boson pair production in e^-e^- collisions, within the framework of the standard model of electroweak interactions. The total cross sections amount to only 1–10 fb for low collider energies, but rise almost linearly with energy beyond $\sqrt{s} = 1$ TeV and reach 200–700 fb at 2 TeV for W^-W^- and W^-Z^0 production. The event rates can then become substantial, and could possibly serve as probes of quartic gauge couplings. However, these processes are unlikely to represent any danger as backgrounds to more exotic reactions which would signal departures from the standard model. Indeed, the final states are characteristic for radiative processes and hence very distinct from exotic pair production without accompanying fermion or final states resulting from the decay of new heavy objects.

After this work was completed we learned of other calculations of the unpolarized total cross sections for the processes considered here [16, 17]. The predictions are in general agreement.

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$e^-e^- \rightarrow$	$W^-W^-\nu_e\nu_e$	$W^-Z^0e^-\nu_e$	$Z^0Z^0e^-e^-$
LL	9.87	23.49	1.30
LR	–	6.95	1.13
RR	–	–	0.57
Unpolarized	2.47	9.35	1.03

Table 1: Total cross sections in femtobarns for various beam polarizations at $\sqrt{s} = 500$ GeV.

observed final state	number of events	exotic reactions
jets	85	$e^-e^- \rightarrow W^-W^-$ [4]
charged leptons + jets	34	
e^- + jets	12	$e^-e^- \rightarrow W^-W^-$ [4] $e^-e^- \rightarrow e^-W^-\nu_e$ [7]
charged leptons	9	
e^-e^-	1	$e^-e^- \rightarrow \tilde{e}^-\tilde{e}^-$ [5, 6] $e^-e^- \rightarrow e^-e^-$ [8]
$\mu^-\mu^-$	1	$e^-e^- \rightarrow X^{--}$ [3] $e^-e^- \rightarrow W^-W^-$ [4] $e^-e^- \rightarrow \tilde{\chi}_1^-\tilde{\chi}_1^-$ [6]

Table 2: Number of background events expected from processes (1-3) to some specific reactions which might signal 'new' physics. Only decay leptons are considered, while the leading leptons are likely to remain unobserved. We assume 10 fb^{-1} integrated luminosity and unpolarized beams at $\sqrt{s} = 500$ GeV.

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